Presupposition and Abduction in Type Theory

Peter Krause*
Institute for Computational Linguistics
University of Stuttgart
Azenbergstr. 12
70174 Stuttgart
Germany
email: krause@ims.uni-stuttgart.de

March 15, 1995

Abstract

This paper is about reasoning with presuppositions in natural language. Presupposition accommodation, as predicted by the linguistic theory of presuppositions as anaphoric expressions, is reconstructed logically as abductive inference in a framework that supports both anaphoric links and a context-dependent notion of propositionhood. Abductive inference arises as a side-effect of the use of the formalism and of characteristics of the communication situation.

The proposal is illustrated by some examples and compared to related approaches.

Keywords: presupposition resolution, abduction, logical frameworks, semantics-pragmatics interface, context-dependence

1 Introduction

In this working note we will study the relation between the presupposition theory of [van der Sandt, 1992] and the type theory of Martin-Löf (MLTT) and its implications for inference processes at the semantics-pragmatics interface. The main claim will be that the anaphoric theory of presuppositions of [van der Sandt, 1992] can be reformulated in MLTT in such a way that presupposition resolution can be understood as abductive inference. To a certain extent, this amounts to a unification of the advantages of the anaphoric theory of presuppositions, of some desirable properties of MLTT as a semantic formalism and the approach of [Hobbs et al., 1993] to semantic interpretation based on abductive

*This note was written when the author was supported by ESPRIT-project DYANA-2. Thanks to David Beaver, Jochen Dörre, Tim Fernando, Esther König, Uwe Mönnich, Aarne Ranta, Robert van Rooy, Antje Roßdeutscher, Michael Schiehlen, Peter Schroeder-Heister and Ede Zimmermann for relevant discussions.

reasoning. It has always been felt that linguistic presuppositions are context dependent phenomena. This intuition has been made precise within the dynamic conception of meaning, where sentences lead to a change of context and presuppositions are viewed as requirements placed by sentences on the contexts they are processed in. This conception has lead to a successful analysis of presupposition projection ([Karttunen, 1974], [Heim, 1983]), which has recently been refined ([van Eijck, 1993], [Zeevat, 1992]). One step further is taken by [van der Sandt, 1992], who identifies presuppositions with the most well-known context-dependent phenomenon, namely anaphora. He claims interesting empirical advantages for his conception of presuppositions as anaphora and shows how it can be implemented within the framework of DRT [Kamp, 1981]. The anaphoric conception of presuppositions and its empirical consequences will be sketched in section 2. Recently, [Ranta, 1994] has analyzed presuppositions as anaphoric expressions in the constructive type theory of [Martin-Löf, 1984]. Natural language texts are translated into type-theoretical contexts. In type theory, propositionhood is context-dependent: whether a propositional expression will be classified as a proposition depends on the context in which it occurs. The type-theoretical analysis of presuppositions is sketched in section 3. In the central section 4, abduction is adapted to type theory, it is shown how abductive inference in presupposition resolution can be explained pragmatically and some examples are given. In section 5, the computational aspects of the mechanism are discussed. In section 6, the proposed account is compared to related approaches and possible applications are discussed.

2 Anaphoric Presuppositions

[van der Sandt, 1992] classifies presupposition triggers as anaphoric expressions and claims that presupposition resolution and anaphora resolution can be treated by the same mechanism. The anaphoric view of presuppositions makes the following empirical predictions:

- Presuppositions can be bound to antecedents:

  (1) John owns a house. The house is old.

  The presupposition triggered by the definite NP in the second sentence is bound to the antecedent indefinite NP in the first sentence.

- Ambiguities can arise through multiple antecedents:

  (2) John owns a house, and so does Bill. The house with the black roof ...

  The presupposition can either be satisfied by John's house or by Bill's house.

- Ambiguities between binding and accommodation are predicted:

  (3) John has a green bike. His old bycicle is still running quite well.

- Pronouns can refer back to antecedents established by accommodation:

(4) His bike is parked in front of the house. It is green.

For further explanations of the empirical predictions of the anaphoric theory of presuppositions, see [van der Sandt, 1992].

3 Presuppositions in Type Theory

For the basic characteristics of MLTT and its application to natural language (in particular its treatment of context-dependence), the reader is referred to [Ranta, 1994, ch. 1-4].
We will largely stick to the terminology from [Ranta, 1994], but we change the notation slightly:

- We use a sequent presentation analoguous to the one used in [Troelstra, 1987]. The context Γ on which a judgment J depends is written to the left of the sequent sign:

  $\Gamma \ \Rightarrow J$

- So-called type premises have been dropped (see [Troelstra, 1987, sect. 2.11]).

For reasons of space, we have to limit ourselves to an explanation of the type-theoretical treatment of definite descriptions and possessives. For a complete overview of those elements of MLTT which are relevant for natural language, see [Ranta, 1994].
Linguistic presuppositions are treated as anaphoric phenomena in type-theoretical grammar. The presuppositions triggered by definite NPs and possessive constructions are reflected by specific term-forming operators Mod and Gen, which are used to formalize the construction triggering the presupposition. The rules for the use of $Mod-$ and $Gen-$ terms tell us in which contexts they may be used.
A definite NP $the_x\ A\ such\ that\ B(x)$ can be formalized in type-theoretical grammar by a so-called Mod-term $Mod(A,(x)B(x),a,b)$.
In our sequent notation, the rules for using the term constructor Mod look as follows:

$$\frac{\Gamma \ \Rightarrow a : A \quad \Gamma, x : A \ \Rightarrow B(x) : prop \quad \Gamma \ \Rightarrow b : B(a)}{\Gamma \ \Rightarrow Mod(A,(x)B(x),a,b) : A}$$

This rule essentially states that in a context which provides for an entity a of the type A corresponding to the head noun of a definite NP and for a proof b that this entity satisfies the content of the modifier of the definite NP, the term $Mod(A,(x)B(x),a,b)$, which formalizes the definite NP, is of type A.

$$\frac{\Gamma \ \Rightarrow a : A \quad \Gamma, x : A \ \Rightarrow B(x) : prop \quad \Gamma \ \Rightarrow b : B(a)}{\Gamma \ \Rightarrow Mod(A,(x)B(x),a,b) \ = \ a : A}$$

This rule states that the value of the Mod - term $Mod(A,(x)B(x),a,b)$ is a. There are no other rules for Mod. Mod-terms can be used for definites modified by adjectives or relative clauses. The definite noun phrase *the black roof* can be formalized as $Mod(roof,(x)black(x),r,p)$, if r and p are a roof and a piece of evidence that that roof is black given in context, respectively. Likewise, the definite NP modified by a relative clause *the man who walks* can be formalized as $Mod(man, (x)walk(x), m, p)$, if m and p are a man and a piece of evidence that that man walks given in context, respectively. The Mod-rules use, as premises, two entities given in context. Any Mod-term is anaphoric to both an entity and a piece of information about that entity. Consequently, it is predicted that definites can only be used properly if both are given in context. In other words, the formalization of definite NPs according to the rules of type-theoretical grammar takes into account their presuppositions. Because definedness of the definite term is necessary for interpretability of the whole sentence, it will not be possible to show that a sentence containing a definite is a proposition if the required arguments of the definite are not present in context. In section 4, it will be shown how such defective contexts can be repaired by introducing background assumptions. Anaphoric genitives are handled by the following rules, which are instances[1] of the rules given in [Ranta, 1994, p. 84]:

$$\frac{\Gamma, x : A, y : B \ \Rightarrow poss(x, y) : prop \quad \Gamma \ \Rightarrow a : A \quad \Gamma \ \Rightarrow b : B \quad \Gamma \ \Rightarrow c : poss(a, b)}{\Gamma \ \Rightarrow Gen(A, B, (x, y)poss(x, y), a, b, c) : B}$$

$$\frac{\Gamma, x : A, y : B \ \Rightarrow poss(x, y) : prop \quad \Gamma \ \Rightarrow a : A \quad \Gamma \ \Rightarrow b : B \quad \Gamma \ \Rightarrow c : poss(a, b)}{\Gamma \ \Rightarrow Gen(A, B, (x, y)poss(x, y), a, b, c) \ = \ b : B}$$

Here a is the possessor, b is the possessee, and c is a witness to the possessor relation. For instance, we can formalize the possessive NP *john's book* by the term $Gen(human, book, (x, y)poss(x, y), john, b, f)$, where b is the book and f the fact that John owns it.

4 Presupposition Resolution and Abduction

In this section, it will be shown how presupposition resolution can be understood as an inferential process using abduction in type theory. First, the notion of abduction will be applied to type theory, resulting in a scheme for abductive reasoning within type theory. Second, an elementary pragmatic mechanism will be sketched which builds on the assumption that both participants use type theory and on basic characteristics of the communication situation. Third, the mechanism will be illustrated by some examples.

4.1 Abduction in Type Theory

Abductive inference in general ([Peirce, 1955]) is explanatory inference. For present purposes, we can codify abductive inference within constructive type theory as follows.
An abductive inference problem is given by a context Γ containing the available background knowledge and an explanandum E, which is an instance of one of the four forms of judgment $A : prop$, $A \ = \ B : prop$, $a : A$, $a \ = \ b : A$.

[1] The arbitrary relation used there has been replaced by a generic possession relation *poss*, which is not to be interpreted as ownership, but as a general relation which is satisfied whenever the participants stand in an appropriate relation which can be expressed by a genitive in the context.

An *explanation* Δ of E with respect to Γ is a type-theoretical context such that for $\Gamma' = \Gamma, \tilde{\Delta}$ we have

1. $\Gamma' \Rightarrow E$.
2. Γ' is consistent.

Abductive inference is the step from some explanandum E in a context Γ to an explanation Δ, leading to an extended context $\Gamma, \tilde{\Delta}$, where the assumptions of the new context are marked by a tilde. This marking serves two purposes. First, the evaluation of the plausibility of a context can depend on which parts had to be assumed and second, it must be possible to tell apart assumed material from asserted material when it comes to non-monotonic revisions.
In order to guide abductive inference and to evaluate its result, a preference order on possible explanations is necessary, which should be able to take into account the cost of assumptions as in weighted abduction ([Hobbs et al., 1993]) and overall plausibility. Let *rank* be a function from type theoretical contexts (marked as indicated) to the non-negative real numbers and define a preference ordering between contexts by

$$\Gamma_1 \leq_{pref} \Gamma_2 \qquad \textit{iff} \qquad rank(\Gamma_1) \leq rank(\Gamma_2)$$

Of course, the precise definition of the preference ordering is a very difficult matter. At the abstract level, we can only say that for any context Γ_0 which is not well-formed we have $rank(\Gamma_0) = 0$, the least preferred value.
An optimal explanation Δ_{opt} of a set $EXPL$ is now recognized by :

$$\forall \Delta \in EXPL \ [rank(\Gamma, \Delta) \leq rank(\Gamma, \Delta_{opt})]$$

The main difference of this way of stating what abductive inference is with the usual formulation for predicate logic is that type - theoretical contexts are ordered.

4.2 Elementary Pragmatics

In this section, the treatment of presupposition resolution by abductive inference is reduced to properties of the type-theoretical formalism and very elementary pragmatic assumptions. We reason informally[2] about the hearer's reconstruction of the speaker's context.

4.2.1 Assumptions

As a general setting, let's consider a pure (text) comprehension situation which is characterized by a cooperative speaker S informing a hearer H by uttering a sequence $T = s_1 \ldots s_n$ of declarative sentences. We assume for simplicity that both S and H behave as though they use type theory as an internal language: The speaker's and the

[2]In order to formalize the argument, an extension of type theory in the direction of a multi-agent system would be required.

hearer's knowledge state are (at least partially) characterized by type-theoretical contexts Γ_S and Γ_H. The shared beliefs of the participants are contained in a common ground $\Gamma_{cg(S,H)}$ such that the contexts of the participants are extensions of it: $\Gamma_{cg(S,H)} \preceq \Gamma_S$ and $\Gamma_{cg(S,H)} \preceq \Gamma_H$. The roles of the participants can be described as follows:

The speaker She selects a subsequence $A_{i1}, \ldots A_{in}$ of the propositions $A_1, \ldots, A_m$ of her context $\Gamma_S = x_1 : A_1 \ldots x_m : A_m$ and translates them into the sentences $s_1, \ldots s_n$ by sugaring. Because she is cooperative, the speaker uses definite descriptions with sufficient descriptive content instead of pronouns whenever the antecedent has not been explicitly introduced or several competing antecedents are available.

The hearer Her goal is optimal comprehension. She tries to reconstruct as much of Γ_S as possible by building a reconstruction $\Gamma_{H(S)}$ of Γ_S incrementally: $\Gamma_{H(S)}$ is the last element of a sequence of provisional reconstructions $\Gamma^1_{H(S)}, \Gamma^2_{H(S)}, \ldots \Gamma^n_{H(S)}$ obtained by interpreting the sentences s_1 up to s_n. The hearer has the capacity to formalize English sentences s_i in type theory using a parsing relation *parse* yielding type-theoretical expressions B. She knows that the speaker uses type theory.

In addition, we assume the following for the sake of simplicity:

1. *parse* is a function. This avoids complications arising from ambiguity.

2. We make the simplifying assumption that $\Gamma^i_{H(S)}$ is correct (although not necessarily complete): none of the judgments from $\Gamma^i_{H(S)}$ needs to be retracted after the update with s_i has been completed.

4.2.2 Consequences

Now we are in a position to show why presupposition resolution corresponds to abductive inferencing in type theory. Consider the situation after the processing of the first i sentences. The hearer's task is to interpret the sentence s_{i+1} in her current model $\Gamma^i_{H(S)}$ of the speaker's context in order to arrive at an updated reconstruction $\Gamma^{i+1}_{H(S)}$. First, she parses s_{i+1}, obtaining a formalization

$$parse(s_{i+1}) = C_{i+1}$$

All the variables in C_{i+1} are chosen distinct from those used in $\Gamma^i_{H(S)}$. From the fact that the speaker uses type theory properly, the hearer can conclude that C_{i+1} is a proposition in the relevant part of the speaker's context, Γ^i_S:

$$\Gamma^i_S \Rightarrow C_{i+1} : prop \tag{1}$$

This has to be explained by making suitable assumptions about the unknown internal context Γ_S, because the hearer's current reconstruction $\Gamma^i_{H(S)}$ of Γ_S might not be sufficient to conclude that $C_{i+1} : prop$. Taking into account the simplifying assumption (2) made above, which assures us that we need not revise destructively the current reconstruction of the speaker's context, we are looking for an optimal extension Δ with the property that $\Gamma^i_{H(S)}, \tilde{\Delta} \;\Rightarrow C_{i+1} : prop$

In other words, the hearer is faced with an abductive inference problem. The explanandum is:

$$C_{i+1} : prop \tag{2}$$

The relevant context is:

$$\Gamma^i_{H(S)}$$

In this way, abductive inference arises naturally from the use of type theory and elementary pragmatic assumptions. The result of the non-deterministic abductive inference step is some $\Gamma^{i'}_{H(S)} \;=\; \Gamma^i_{H(S)}, \tilde{\Delta}$, an extension of $\Gamma^i_{H(S)}$ by a minimal explanation $\tilde{\Delta}$ such that $\Gamma^{i'}_{H(S)} \;\Rightarrow C_{i+1} : prop$. The hearer can now assume a witness for the new proposition to get the new model

$$\Gamma^{i+1}_{H(S)} \;=\; \Gamma^{i'}_{H(S)}, z : \tilde{C}_{i+1}$$

4.3 Examples

Below, examples for the two type-theoretical constructors discussed in 3 are discussed.

4.3.1 Definite descriptions

Two examples illustrate the above mechanism: First, consider the following sentence uttered in the empty context $\Gamma^0_{H(S)}$:

(5) The tall man is approaching.

The hearer H parses the sentence:

$$approach(Mod(man, (x)tall(x), m, f))$$

The two preconditions for propositionhood are accommodated in the global context by abductive inference:

$$\Gamma^{0'}_{H(S)} \;=\; m : \widetilde{man}, f : \widetilde{tall}(m)$$

This explains the propositionhood of the dependent proposition:

$$\Gamma^{0'}_{H(S)} \;\Rightarrow approach(Mod(man, (x)walk(x), m, f)) : prop$$

Finally, the discourse model is incremented:

$$\Gamma^1_{H(S)} \;=\; m : \widetilde{man}, f : \widetilde{walk}(m), z : approach(Mod(man, (x)tall(x), m, f))$$

In the following example of an epithetic use of a definite description, part of the presupposition is found in the context, another part of it is introduced during abductive inference.

(6) John owns a house. The old villa is in a garden.

In this case, the sentence s_1 will be parsed as[3]

$$(\Sigma x_1 : house)own(Def(human, john), x_1)$$

Here the proper name has been treated like a definite noun phrase[4]. The hearer reconstructs a context $\Gamma^0_{H(S)}$ where $john$ is a human and which allows us to prove that the first sentence is a proposition in that context:

$$\Gamma^{0'}_{H(S)} = \widetilde{john : human} \Rightarrow (\Sigma x_1 : house)own(Def(human, john), x_1) : prop$$

She can increment this intermediate context to arrive at

$$\Gamma^1_{H(S)} = \widetilde{john : man},\ z_1 : (\Sigma x_1 : house)own(Def(human, john), x_1)$$

The sentence s_2 is parsed as

$$(\Sigma x_2 : garden)in(Mod(villa, (x)old(x), v, f), x_2)$$

Let's assume that the lexical semantics which is part of the common ground $\Gamma_{cg(S,H)}$ contains a meaning postulate (3) saying that a villa is nothing else but a beautiful house.

$$villa =_{def} (\Sigma x : house)beautiful(x) \tag{3}$$

So the hearer can rewrite the result of parsing in the following way:

$$(\Sigma x_2 : garden)in(Mod((\Sigma x_3 : house)beautiful(x_3), (x)old(x), v, f), x_2)$$

This propositional expression will be a proposition in every context which provides for the variables v and f. By reasoning forwards from the required assertion $v : (\Sigma x_3 : house)beautiful(x_3)$ using the Σ-elimination rule, the hearer can find out that a house and witnessing information that that house is beautiful are needed. There is a contextually given house. So it is sufficient to assume its beauty (f_1) and its old age (f_2):

$$\begin{aligned}\Gamma^{1'}_{H(S)} = {} & \widetilde{john : man}, z_1 : (\Sigma x_1 : house)own(Def(human, john), x_1), \\ & f_1 : \widetilde{beautiful(p_1(z_1))}, f_2 : \widetilde{old(p_1(z_1))}\end{aligned}$$

In this context, she can perform Σ - introduction:

[3] Σ is the type-theoretical counterpart of both conjunction and existential quantification. The rules for Σ-introduction and Σ-elimination which are used below are as follows:

$$\frac{a : A \quad b : B(a)}{\langle a, b\rangle : (\Sigma x : A)B(x)}\ \Sigma I \qquad \frac{z : (\Sigma x : A)B(x)}{p_1(z) : A}\ \Sigma E \qquad \frac{z : (\Sigma x : A)B(x)}{p_2(z) : B(p_1(z))}\ \Sigma E$$

[4] Note that the assumption that proper names are introduced at the top level familiar from discourse representation theory is explained by the abductive mechanism.

$$\Gamma^{1''}_{H(S)} = \Gamma^{1'}, \langle p_1(z_1), f_1\rangle : (\Sigma x : house)beautiful(x)$$

Finally, the presupposed variables are identified with contextually given objects:

$$\begin{aligned}\Gamma^{1'''}_{H(S)} = {} & john : man, z_1 : (\Sigma x_1 : house)own(Def(human, john), x_1), f_1 : beautiful(p_1(z_1)),\\ & f = f_2 : old(p_1(z_1)), v = \langle p_1(z_1), f_1\rangle : (\Sigma x : house)beautiful(x)\end{aligned}$$

In this context, s_2's formalization is a proposition:

$$\Gamma^{1'''}_{H(S)} \Rightarrow (\Sigma x_2 : garden)in(Mod(villa, (x)old(x), v, f), x_2) : prop$$

The result of updating with s_2 is thus

$$\begin{aligned}\Gamma^{2}_{H(S)} = {} & john : man, z_1 : (\Sigma x_1 : house)own(Def(human, john), x_1), f_1 : beautiful(p_1(z_1)),\\ & f = f_2 : old(p(z_1)), v = \langle p_1(z_1), f_1\rangle : (\Sigma x : house)beautiful(x),\\ & z_2 : (\Sigma x_2 : garden)in(Mod(villa, (x)old(x), v, f), x_2)\end{aligned}$$

4.3.2 Possessive noun phrases

In order to illustrate the *Gen* constructor used for formalizing possessives, let's consider a simple example which shows that presupposition triggers can be recursively embedded:

(7) John drives his neighbour's car.

This sentence can be formalized as

$$\begin{aligned}C_1 = {} & drive(Def(human, john), Gen(man, car, (x, y)poss(x, y),\\ & Gen(human, human, (x, y)poss(x, y), j, n, c_1), d, c_2))\end{aligned}$$

Here *poss* is an unspecified possessor relation. The result of reconstructing a context $\Gamma^1_{H(S)}$ in which this expression is a proposition ($\Gamma^1_{H(S)} \Rightarrow C_1 : prop$) is:

$$\Gamma^1_{H(S)} = john : human, n : human, c_2 : poss(john, n), d : car, c_1 : poss(n, d)$$

Now it is possible to apply the abductive procedure for resolving underspecified relations used by [Hobbs et al., 1993] for the interpretation of compound nominals to arrive at a context equivalent with:

$$\Gamma^1_{H(S)} = john : human, n : human, c_2 : lives\text{-}next\text{-}to(john, n), d : car, c_1 : own(n, d)$$

Axioms are needed which say that the specific relations of *living-next-to* between persons and their neighbours and of *owning* between persons and their cars imply the underspecified possessor relation *poss*. We only point at this possibility to couple abductive reasoning with explananda of the form $x : A$ with the special kind of abductive reasoning needed for resolving presuppositions, where the explanandum is of the form $A : prop$. But other approaches to the determination of contextually appropriate possessor relations are compatible with the abductive resolution of presuppositions as well.

4.4 Possible Effects of the Abductive Inference Step

Abductive inference for presupposition resolution can have different results. In the simplest case, an anaphoric element is merely identified with some element present in the context by adding an equation to the context. The presupposition can also be justified by applying elimination rules to the context in order to construct some element of the required type(s) which then can be identified with the anaphoric term. These two cases correspond to presupposition verification.

In presupposition verification, no new information is added to the context. Another option is the introduction of a new assumption covering all the presupposed material. This corresponds to presupposition accommodation. And finally, it is possible to mix accommodation and contextual identification if some part of the presupposition is already present in the context, whereas some other part has to be introduced during abductive reasoning. All these cases are subsumed by the general concept of *presupposition justification* by [Kamp and Roßdeutscher, 1994, p. 208]. Note that in the case of contextual identification (or presuppositional binding) different antecedent expessions can be selected, just as in [van der Sandt, 1992]. All the basic empirical predictions mentioned in section 2 are therefore shared by our account. The unified mechanism of abductive inference can predict a large variety of superficially different phenomena.

5 Computational Aspects

The main point of this note is that the resolution of anaphoric presuppositions can be logically understood as abductive inference. This immediately raises the question whether an efficient implementation of the mechanism is possible. A comprehensive answer to this question is beyond the scope of this note. Although the computational complexity of arbitrary abductive reasoning in constructive type theory is obviously high, there are also a number of reasons to believe that abductive presupposition resolution is practically feasible.

First of all, general methods for making abduction efficient ([Hobbs et al., 1993], [Kakas et al., 1994]) can probably adapted to the MLTT setting.[5] Second, it seems as though forward chaining from intermediate goals as used in the second example of section 4.3.1 can guide abductive reasoning towards minimal explanations: by quickly identifying which parts of the presupposed information are already present in context, the need for abductive accommodation can be minimized. Third, some of the linguistic research on preferences for accommodation can be used to formulate heuristics. Fourth, the natural language application of type theory relies on a proper subset of the rules of type theory. Finally, many examples discussed in the linguistic literature can be treated with a proper subset of the inference rules used in the linguistic application of type theory.

[5] A detailed analysis would be necessary to see which methods would be most promising.

6 Summary

Because this note attempts to contribute to a unification of the advantages of different approaches to presuppositional reasoning, in the first part of this summary the gains of the treatment are evaluated from the different points of views which have been compared.

6.1 Evaluation from different perspectives

6.1.1 Anaphoric presuppositions

The formulation of the anaphoric conception of presuppositions by [van der Sandt, 1992] and the above proposal differ in the following respects:

- MLTT provides "discourse referents" representing not only individuals as in DRT, but also inhabitants of logically complex propositions. In a judgment $x : A$, A can be a logically complex proposition, as well as a basic category like *man*. This feature of the formalism enables us to avoid the complications of introducing an additional device such as the "anaphoric structures" (α-DRSs) of [van der Sandt, 1992].

- The fact just mentioned leads to a completely uniform resolution procedure. Presupposition resolution becomes even more analoguous to anaphora resolution than in the theory of [van der Sandt, 1992]: both mechanisms consist in identifying a free variable in the representation of the current sentence with some object given in context.

- The formulation of presupposition resolution in type theory shows that it can be understood as an inference process. In that sense, the type theoretical treatment of presuppositions can be seen as a *logical reconstruction* of van der Sandt's theory. In [van der Sandt, 1992], presupposition resolution is rather a copying operation on representations which in itself is not an inference process, although it is constrained by inference via the constraints on contextual acceptability (see [van der Sandt, 1992, p. 367]).

- No separate module for presupposition resolution is needed in a natural-language system based on MLTT, if the system contains a general module for abductive reasoning.

- In section 4.2 it was shown how presuppositional inferences arise from pragmatic considerations.

- The anaphoric conception of presupposition is reconciled with the view that presuppositional satisfaction amounts to logical entailment, because in MLTT, anaphoric binding and entailment coincide. In particular, the notion of entailment of MLTT between propositions is always relative to a choosen proof.

6.1.2 DRT

We have (partially) reconstructed the anaphoric analysis of presuppositions which is widely used in DRT ([Kamp and Rossdeutscher, 1992], [van der Sandt, 1992]) in a logical system which supports some essential features of DRT ([Ahn and Kolb, 1990]). Maybe this reconstruction suggests to actually formulate a version of DRT with a context-dependent notion of propositionhood. The reconstruction has emphasized the inferential aspects. Of course, only part of the lexically driven inferences in [Kamp and Rossdeutscher, 1992] can be captured with the rules formulated above. For a more complete treatment, temporal presupposition triggers would have to be taken into account.

6.1.3 Natural language semantics in MLTT

With respect to natural language semantics in MLTT, it was shown above that the introduction of background hypotheses in order to make sense of a text discussed in [Ranta, 1994, sect. 6.6] can be understood as abductive reasoning. The comparison with [van der Sandt, 1992] shows the relevance of empirical work from DRT for type-theoretical grammar. One of the consequences of this comparison is that empirical problems for the anaphoric theory of [van der Sandt, 1992] are likely to become empirical problems for type-theoretical grammar as well. In section 4.2 it was discussed how the contextual resolution of presuppositions can be seen as an epiphenomenon of the fact that the participants use type theory. It remains to be seen in how far other pragmatic phenomena can be reconstructed in type theory and which extensions of the formal apparatus are needed for them.

6.1.4 Abductive reasoning in NLP

[Hobbs et al., 1993] have proposed to use abductive reasoning in natural language processing for a wide variety of interpretation problems. Among other phenomena, they consider definite reference, in particular, cases of anaphoric definite descriptions.
They use first-order logic as their representation formalism. However, there are two reasons that first-order logic cannot be an adequate medium for resolving presuppositions by abduction.

Reason 1 First-order predicate calculus is not adequate for dealing with anaphoric connections in discourse

In formal semantics it is widely acknowledged that first-order predicate calculus is not an adequate formalism for representing intersentential anaphoric connections.[6] For a proper treatment of anaphoric binding accross sentence boundaries, an appropriate formalism such as discourse representation theory [Kamp, 1981], dynamic predicate logic [Groenendijk and Stokhof, 1991] or constructive type theory [Ranta, 1994] is indispensible. It therefore seems unlikely that the informal treatment of definite noun phrases from [Hobbs et al., 1993] can be made precise without moving to another semantic formalism.

[6] See, e.g., [Gamut, 1990] for the reasons.

Reason 2 In first-order predicate logic, the relevant explanandum cannot be expressed.

> In section 4 it was emphasized that the underlying explanandum was a judgment $A : prop$, where A is a propositional expression. This explanandum had to be explained with respect to the current reconstruction of the speaker's context. It is obvious that such judgments can not be expressed in predicate logic, because syntactic wellformedness (propositionhood) is a meta-level notion in FOL.

These reasons suggest that the analysis of presuppositions by abductive reasoning can be improved by using a formalism which supports both anaphoric links and a context-dependent notion of propositionhood.
The above treatment seems to be a good candidate for a framework for an account of presupposition resolution which is based on a preference ordering between common sense theories and overcomes a purely structural conception of accommodation as argued for on empirical grounds by [Beaver, 1994].
Preference orderings between possible explanations lie at the heart of abductive reasoning, while the construction of explanations in constructive type theory can flexibly use inference rules to split up and collect contextual information from different places.
Abductive reasoning in NLP can be used for resolving presuppositions, if an appropriate formalism is used, which supports both anaphoric links in texts and a context-dependent notion of proposition expressible within the language and which in addition, has a suitable proof theory. MLTT has these properties, but any other formalism satisfying this criterion would do as well. The possibility to express anaphoric links is important to be able to properly represent the outcome of contextual reasoning, the context dependent notion of proposition is important to formulate the explanandum of the abductive inference problem.

6.1.5 Semantics-pragmatics interface

In DRT and MLTT, presupposition failure can be seen as a failure of semantic interpretability. In the analysis outlined in this paper, failure of semantic interpretability and reasoning about the other participant's context gave rise to a process of abductive inference which can be classified as pragmatic. Semantics and pragmatics are tightly coupled because the pragmatic reasoning is carried out in the same formalism as semantic interpretation.

6.2 Possible Applications

The use of type theory as a semantic representation formalism opens up the possibility to construct natural language interfaces for logical frameworks without detours. A mechanism which handles presuppositions can help to improve the coverage of and ease of interaction with such interfaces, because people often use presuppositional expressions to convey information in an implicit way. Variables can be declared "on the fly" at the same time when they are used. The fact that presupposition resolution in type-theory can be seen as abductive *inference* can simplify the architecture of NLP systems, because the need for a specific resolution module is at least reduced.

6.3 Present limitations and future work

Some aspects have not yet been discussed in this note and point to the need for further work.
The uniqueness presuppositions associated with definite NPs have not yet been taken into account. See [Ranta, 1994, sect. 4.7] for a discussion. It remains to be seen how the mechanism sketched in section 4.2 squares with the literature on multi-agent reasoning, and how it can be formalized best. There is some similarity to approaches which mix the object-language and the meta-language in order to reason about knowledge and belief such as [Kowalski and Kim, 1991]. In this paper, local and intermediate accommodation have not yet been taken into account for two reasons. Firstly, these mechanisms are still much debated from an empirical point of view. Secondly, a proper account within type theory is only possible if the interaction between syntax and semantics is less sequential than in the simplified picture of section 4.2. The above approach inherits all empirical problems of [van der Sandt, 1992] because it is essentially a re-implementation of the main ideas of [van der Sandt, 1992] in type theory. For a wider applicability, the approach must be integrated into a theory of dialogue.

References

[Ahn and Kolb, 1990] Ahn, R. and Kolb, H. (1990). Discourse Representation meets Constructive Mathematics. Technical Report 16, ITK, Tilburg.

[Beaver, 1994] Beaver, D. (1994). A Plea for Common Sense. In *Proceedings of the 5th Hungarian Symposium on Logic and Language.*

[Gamut, 1990] Gamut, L. (1990). *Logic, Language and Meaning.* University of Chicago Press.

[Groenendijk and Stokhof, 1991] Groenendijk, J. and Stokhof, M. (1991). Dynamic Predicate Logic. *Linguistics and Philosophy*, 14:39–100.

[Heim, 1983] Heim, I. (1983). On the projection problem for presuppositions. In *Proceedings of the West Coast Conference on Formal Linguistics*, volume 2, pages 114–126.

[Hobbs et al., 1993] Hobbs, J., Stickel, M., Appelt, D., and Martin, P. (1993). Interpretation as Abduction. *Artificial Intelligence*, 63:69–142.

[Kakas et al., 1994] Kakas, A., Kowalski, R., and Toni, F. (1994). Abductive logic programming. *Journal of Logic and Computation*, 2(6):719–770.

[Kamp, 1981] Kamp, H. (1981). A Theory of Truth and Semantic Representation. In Groenendijk, J. and Stokhof, M., editors, *Formal Methods in the Study of Language.* Mathematical Centre, Amsterdam.

[Kamp and Rossdeutscher, 1992] Kamp, H. and Rossdeutscher, A. (1992). Remarks on Lexical Structure, DRS Construction and Lexically Driven Inferences. Arbeitspapiere des Sonderforschungsbereichs 340: Sprachtheoretische Grundlagen der Computerlinguistik 21, Universität Stuttgart.

[Kamp and Roßdeutscher, 1994] Kamp, H. and Roßdeutscher, A. (1994). DRS - Construction and Lexically Driven Inference. *Theoretical Linguistics*, 20(2/3):165–235.

[Karttunen, 1974] Karttunen, L. (1974). Presupposition and linguistic context. *Theoretical Linguistics*, 1:181–94.

[Kowalski and Kim, 1991] Kowalski, R. and Kim, J.-S. (1991). A Metalogic Programming Approach to Multi-Agent Knowledge and Belief. In Lifschitz, V., editor, *Artificial Intelligence and Mathematical Theory of Computation*, pages 231–246. Academic Press.

[Martin-Löf, 1984] Martin-Löf, P. (1984). *Intuitionistic Type Theory*. Studies in Proof Theory. Bibliopolis, Napoli.

[Peirce, 1955] Peirce, C. S. (1955). Abduction and induction. In Buchler, J., editor, *Philosophical Writings of Peirce*, pages 150–156. Dover Books, New York.

[Ranta, 1994] Ranta, A. (1994). *Type-Theoretical Grammar*. Oxford University Press.

[Troelstra, 1987] Troelstra, A. (1987). On the Syntax of Martin-Löfs Type Theories. *Theoretical Computer Science*, 51:1–26.

[van der Sandt, 1992] van der Sandt, R. (1992). Presupposition projection as anaphora resolution. *Journal of Semantics*, 9:333–377.

[van Eijck, 1993] van Eijck, J. (1993). The Dynamics of Description. *Journal of Semantics*.

[Zeevat, 1992] Zeevat, H. (1992). Presupposition and accommodation in update semantics. *Journal of Semantics*, 9:379–412.